Oscillator decomposition of time series data

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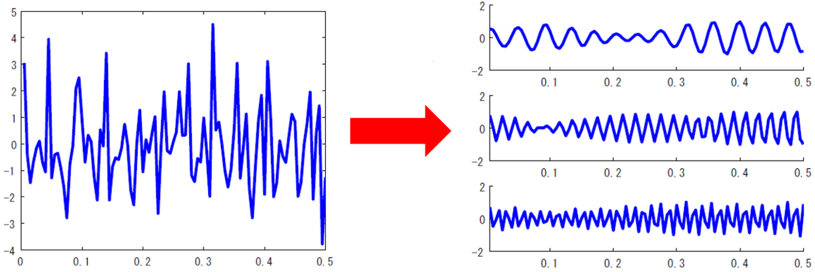
Abstract

- We develop a method for decomposing time series data into oscillators (without band-pass filtering).



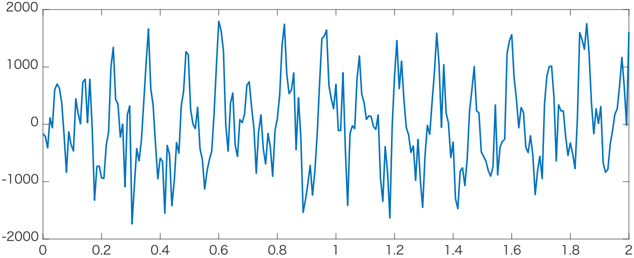
- paper: Matsuda and Komaki (2017a&b, *Neural Computation*)

Background: neural oscillation

Neural oscillation

- neural time series: recording of brain activity
  - EEG, MEG, fMRI, fNIRS
- They are composed of several oscillation components.
  - delta: 0.5-4 Hz, alpha: 8-13 Hz, beta: 13-20 Hz

- Local Field Potential (LFP) from rat hippocampus



- Theta component (6-10 Hz) is clearly seen.

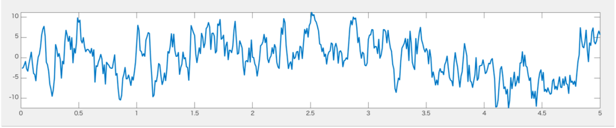
Phase of neural oscillation

- Phase of neural time series plays an important role in neural information processing.

- phase reset occurs in response to external stimuli. (Makeig et al., 2012; Lopour et al., 2013)
- theta phase precession in hippocampal LFP encodes the place. (O'Keefe and Recce, 1993)
- phase synchronization is important for coordination of distant neural assemblies. (Buzsaki, 2011)

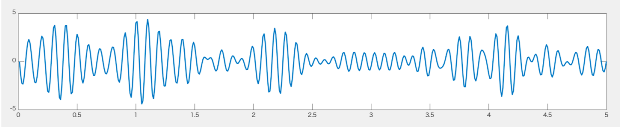
Conventional analysis with band-pass filtering

raw EEG

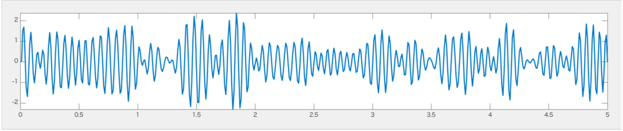


band-pass filtering

alpha (8-13 Hz)



beta (13-20Hz)



Hilbert transform

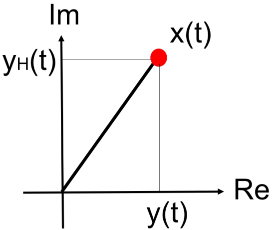
alpha phase, beta power, ...

Hilbert transform

$$y(t) \mapsto y_H(t) = \int_{-\infty}^{\infty} \frac{y(\tau)}{t-\tau} \mathrm{d}\tau$$

- analytical signal (complex time series)

$$x(t) = y(t) + i\, y_H(t)$$



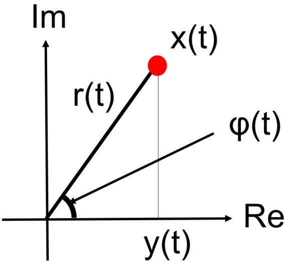
- In frequency domain,

$$X(\omega) = \begin{cases} 2Y(\omega) & (\omega > 0) \\ Y(\omega) & (\omega = 0) \\ 0 & (\omega < 0) \end{cases}$$

Hilbert transform

- The phase $\phi(t)$ and amplitude $r(t)$ are defined by the angle and absolute value of $x(t)$.

$$\phi(t) = \arg x(t), \quad r(t) = |x(t)|$$



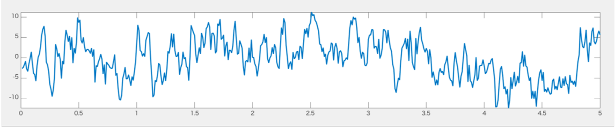
- When $y(t) = \cos \omega t$,

$$y_H(t) = \sin \omega t, \quad x(t) = \exp(i\omega t)$$

$$\phi(t) = \omega t, \quad r(t) = 1$$

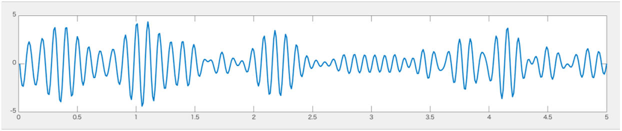
Conventional analysis with band-pass filtering

raw EEG

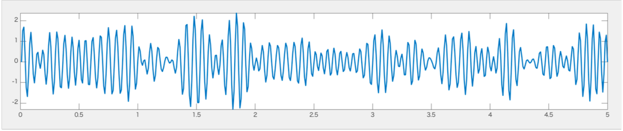


band-pass filtering

alpha (8-13 Hz)



beta (13-20Hz)



Hilbert transform

alpha phase, beta power, ...

Limitations of band-pass filtering

- requires subjective selection of filters
  - definition of frequency band varies among studies..
  - alpha = 8-13 Hz ? 9-12 Hz ?
  - alpha/beta frequency may depend on individuals
- does not account for measurement noise
- distorts the waveform
  - Siapas et al. (2005), methods section
  - since the decomposition is exact even when the input contains a mixture of signal and noise, both enter the instantaneous phase and amplitude components, and thus any denoising must occur at the earlier filtering step.
  - This dictates the use of narrow band filters in conjunction with this method that can distort the input signal waveform.

→ We overcome them by state space model approach

Oscillator decomposition

State space model

- general framework for estimating hidden dynamics from time series data

$$
\begin{aligned}
x_{t+1} &= f(x_t, v_t), \quad v_t \sim p(v_t) \\
y_t &= h(x_t, w_t), \quad w_t \sim p(w_t)
\end{aligned}
$$

- x_t: state (unobserved), y_t: data (observed)
- The posterior $p(x_s \mid y_1, \cdots, y_t)$ can be computed sequentially
  - filtering $(s = t)$, smoothing $(s < t)$, prediction $(s > t)$

Gaussian linear state space model

- Special class of state space models

$$
\begin{aligned}
x_{t+1} &= F x_t + G v_t, \quad v_t \sim \mathrm{N}(0, Q) \\
y_t &= H x_t + w_t, \quad w_t \sim \mathrm{N}(0, R)
\end{aligned}
$$

- The posteriors $p(x_s \mid y_1, \cdots, y_t)$ are Gaussian and they are obtained by matrix computation (Kalman filter, Kalman smoother).

cf. Bayesian seasonal adjustment

- State space models can be used to decompose time series
- Kitagawa and Gersch (1984) developed a method for decomposing economic time series into trend and seasonal components based on a Gaussian linear state space model.

$$T_{t+1} = F_T T_t + G_T v_t, \quad S_{t+1} = F_S S_t + G_S v_t, \quad v_t \sim \mathrm{N}(0, Q)$$

$$y_t = T_t + S_t + w_t, \quad w_t \sim \mathrm{N}(0, R)$$

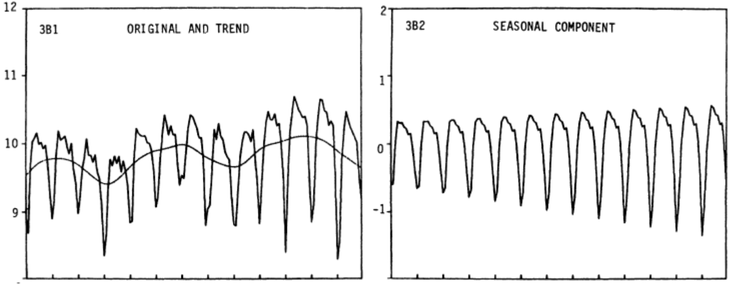
- decomposition is obtained by smoothing

$$y_t = T_{t|N} + S_{t|N} + w_{t|N}$$

$$T_{t|N} = \mathrm{E}[T_t \mid Y_1, \cdots, Y_N], \quad S_{t|N} = \mathrm{E}[S_t \mid Y_1, \cdots, Y_N]$$

cf. Bayesian seasonal adjustment

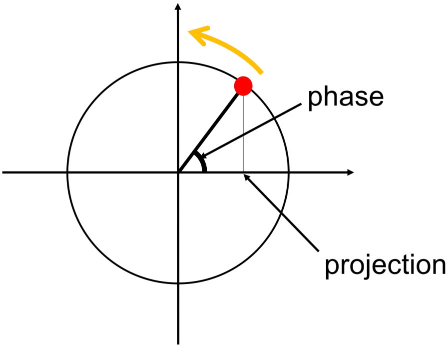
- Construction Housing Starts North data
  - Kitagawa and Gersch (1984)



- Seasonal adjustment improves prediction accuracy.

Model building

- assumption: several oscillators underlie the time series data
- Each oscillator rotates on a 2-dim. plane with fluctuation.



- The observed time series is regarded as the sum of the projections of the oscillators (plus noise).
  - trigonometric function = projection of circular motion
  - cf. Hilbert transform

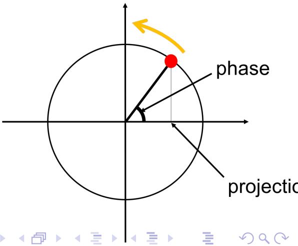
System model for oscillators

- We describe each oscillator by a Gaussian linear model
  - Δt: sampling period

$$\begin{pmatrix} x_{t+1,1} \\ x_{t+1,2} \end{pmatrix} \sim \mathrm{N}\left(a \begin{pmatrix} \cos(2\pi f \Delta t) & -\sin(2\pi f \Delta t) \\ \sin(2\pi f \Delta t) & \cos(2\pi f \Delta t) \end{pmatrix} \begin{pmatrix} x_{t,1} \\ x_{t,2} \end{pmatrix}, \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix} \right)$$

- f: frequency, a: regularity $(0 < a < 1)$, σ^2: power
- In each time step, the oscillator rotates through an angle $2\pi f \Delta t$ with fluctuation.
- At each time point, the phase is defined as

$$\phi_t = \tan^{-1} \frac{x_{t,2}}{x_{t,1}}$$



State space model for oscillator decomposition

- K: number of oscillators

$$\begin{pmatrix} x_{t+1,1}^{(k)} \\ x_{t+1,2}^{(k)} \end{pmatrix} \sim \mathrm{N}\left(a_k \begin{pmatrix} \cos(2\pi f_k \Delta t) & -\sin(2\pi f_k \Delta t) \\ \sin(2\pi f_k \Delta t) & \cos(2\pi f_k \Delta t) \end{pmatrix} \begin{pmatrix} x_{t,1}^{(k)} \\ x_{t,2}^{(k)} \end{pmatrix}, \begin{pmatrix} \sigma_k^2 & 0 \\ 0 & \sigma_k^2 \end{pmatrix} \right)$$

$$(k = 1, \cdots, K)$$

$$y_t \sim \mathrm{N}\left(\sum_{k=1}^{K} x_{t,1}^{(k)}, \tau^2 \right)$$

- Based on the observation $y_1, \cdots, y_T$, the posterior of x_t is calculated by the Kalman smoother.

  $\rightarrow$ decomposition of y_t into $x_{t,1}^{(1)}, \cdots, x_{t,1}^{(K)}$ is obtained

- The phase of each oscillator is also estimated (with credible interval).

parameter estimation & model selection

- Model parameters $\theta_K = (f_1, \cdots, f_K, a_1, \cdots, a_K, \sigma_1^2, \cdots, \sigma_K^2, \tau^2)$ and number of oscillators K are determined in a data-driven manner.

- Model parameters are estimated by empirical Bayes method.
  - maximization of marginal likelihood

$$\hat{\theta}_K = \arg\max_{\theta_K} \log p(y \mid \theta_K)$$

- Number of oscillators are determined by minimizing Akaike Bayes Information Criterion (ABIC).

$$\mathrm{ABIC}_K = -2 \log p(y \mid \hat{\theta}_K) + 2(3K + 1)$$

$\rightarrow$ natural decomposition of time series data is attained

Remark: initial value setting by AR model

- We select the initial value for parameter estimation by using the AR model
- AR model can be viewed as a sum of oscillation components
  - each characterstic root z represents an oscillator with frequency $\arg z$
- examine the location and sharpness of each peak in the fitted AR spectrum
  $\rightarrow a_k, f_k$
- fit spectrum to periodogram based on the Whittle likelihood
  $\rightarrow \sigma_k^2$

Simulation results

Simulation setting

- We generated time series with three oscillators from the following model.
  - data length: 1000
  - sampling frequency: 200 Hz
  - 25 Hz oscillator & 50 Hz oscillator & 75 Hz oscillator

$$\log r_{t+1}^{(k)} \sim \mathrm{N}(b_k \log r_t^{(k)}, 0.1^2) \quad (k = 1, \cdots, K)$$

$$\phi_{t+1}^{(k)} \sim \mathrm{vM}(\phi_t^{(k)} + \Delta\phi_k, \kappa_k) \quad (k = 1, \cdots, K)$$

$$y_t \sim \mathrm{N}\left(\sum_{k=1}^{K} r_t^{(k)} \cos \phi_t^{(k)}, \tau^2\right)$$

- $\mathrm{vM}(\mu, \kappa)$ is the von Mises distribution with center μ and concentration κ.

parameter estimation & model selection

- ABIC takes minimum at $K = 3$.

  → The proposed method selects the correct number of oscillators

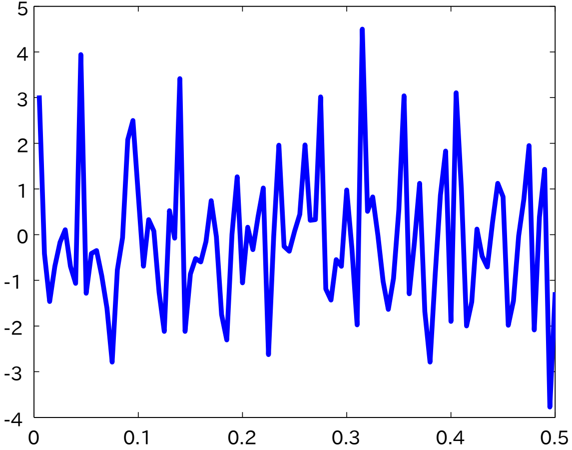
- parameter estimates for $K = 3$

$$\hat{f}_1 = 25.22, \hat{f}_2 = 50.40, \hat{f}_3 = 75.10$$

→ The proposed method estimates the frequencies accurately

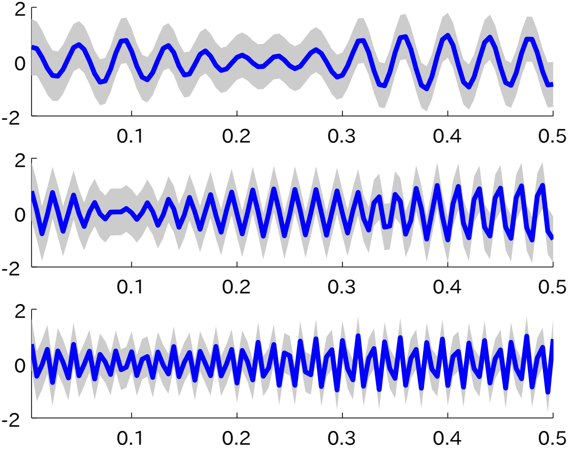
decomposition

- original time series



decomposition

- obtained three oscillation components
  - with 67 % credible intervals



phase estimation

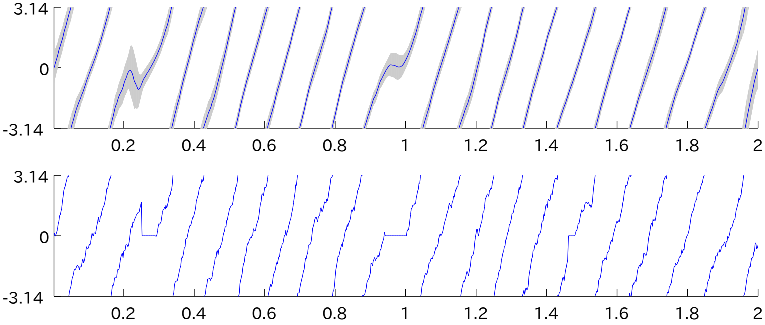
- mean squared errors of phases $\phi_t^{(1)}, \phi_t^{(2)}, \phi_t^{(3)}$ (in radian)

	$\phi^{(1)}$	$\phi^{(2)}$	$\phi^{(3)}$
proposed	0.23	0.16	0.13
Hilbert	0.27	0.37	0.48

- The proposed method has better accuracy in phase estimation.
  - Hilbert transform uses band-pass filters with passband 20-30 Hz, 40-60 Hz, and 60-90 Hz.

phase reset detection

- time series with two oscillators (10 Hz & 30 Hz)
  - several phase resets in 10 Hz oscillator
- It was decomposed into 9.78Hz and 29.68Hz oscillators.
- upper: estimated phase, lower: true phase



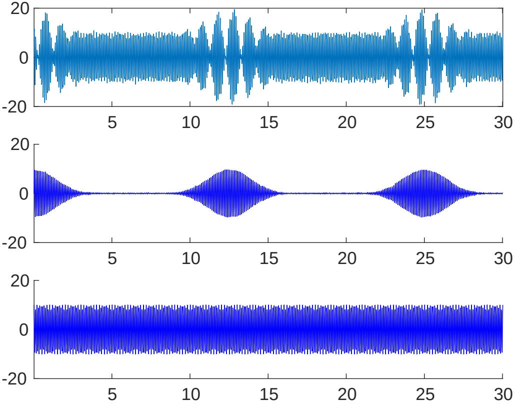
- The proposed method detects phase resets succeesfully.

ripple detection

- time series with ripples (intermittent oscillations)

$$g(t) = A\cos^8(2\pi f_0 t)\sin(2\pi f_1 t) + 10\cos(2\pi f_2 t)$$

- decompostion result

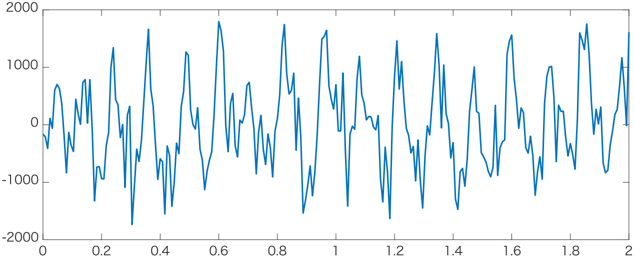


- The proposed method detects ripples successfully.

Application to real data

Application to hippocampal LFP

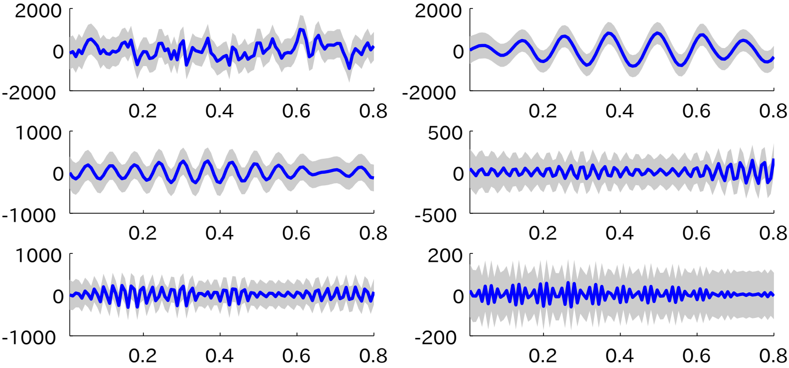
- Local Field Potential from rat hippocampus
  - Mizuseki et al. (2009)
- sampling frequency: 125 Hz
- data length: 250 (two seconds)



Decomposition (hippocampal LFP)

- decomposed into six oscillators

$$\hat{f}_1 = 6.45, \hat{f}_2 = 8.00, \hat{f}_3 = 15.79, \hat{f}_4 = 34.63, \hat{f}_5 = 40.35, \hat{f}_6 = 55.44$$



- Their phases develop regularly.

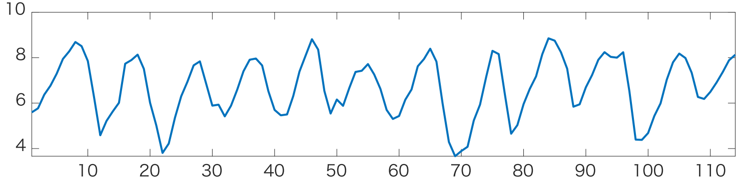
Interpretation (hippocampal LFP)

- Two oscillators (6.45 Hz, 8.00 Hz) correspond to the theta rhythm.
  - Band-pass filtering with theta band cannot separate these oscillators.
- Another oscillator (15.79 Hz) corresponding to the alpha rhythm likely exists.
- Higher frequency oscillators are also evident.
  - Future research may investigate the role of these oscillators, including the potential for their phases to encode internal information.

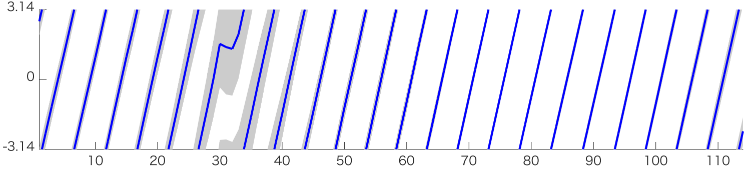
- In this way, neural oscillators underlying the hippocampal LFP are extracted in a data-driven manner.

Application to Canadian Lynx data

- annual number of the Canadian lynxes (1821 - 1934)



- Among six oscillators, one oscillator (5 years period) showed phase reset around 1850.

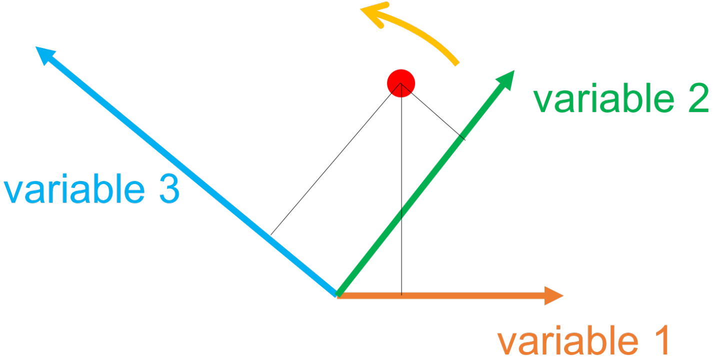


- In this way, the phase dynamics may provide an interesting insight from time series data.

Extension to multivariate time series

Extension to multivariate time series

- Several oscillators underlie multivariate time series.
  - neural oscillators & electrodes
  - earthquake origins & stations
- model assumption: projection of each oscillator superposes on each variable.
  - projection describes amplitude & phase modulation



State space model

$$\begin{pmatrix} x_{t+1,1}^{(k)} \\ x_{t+1,2}^{(k)} \end{pmatrix} = a_k \begin{pmatrix} \cos(2\pi f_k \Delta t) & -\sin(2\pi f_k \Delta t) \\ \sin(2\pi f_k \Delta t) & \cos(2\pi f_k \Delta t) \end{pmatrix} \begin{pmatrix} x_{t,1}^{(k)} \\ x_{t,2}^{(k)} \end{pmatrix} + \begin{pmatrix} v_{t,1}^{(k)} \\ v_{t,2}^{(k)} \end{pmatrix},$$

$$y_{t,j} = \sum_{k=1}^{K} (c_{jk,1} x_{t,1}^{(k)} + c_{jk,2} x_{t,2}^{(k)}) + w_{t,j},$$

$$(v_{t,1}^{(k)}, v_{t,2}^{(k)})^\top \sim \mathrm{N}_2(0, \sigma_k^2 I),$$

$$(w_{t,1}, \cdots, w_{t,J})^\top \sim \mathrm{N}_J(0, \tau^2 I).$$

System model

- same with the univariate case

$$\begin{pmatrix} x_{t+1,1}^{(k)} \\ x_{t+1,2}^{(k)} \end{pmatrix} = a_k \begin{pmatrix} \cos(2\pi f_k \Delta t) & -\sin(2\pi f_k \Delta t) \\ \sin(2\pi f_k \Delta t) & \cos(2\pi f_k \Delta t) \end{pmatrix} \begin{pmatrix} x_{t,1}^{(k)} \\ x_{t,2}^{(k)} \end{pmatrix} + \begin{pmatrix} v_{t,1}^{(k)} \\ v_{t,2}^{(k)} \end{pmatrix},$$

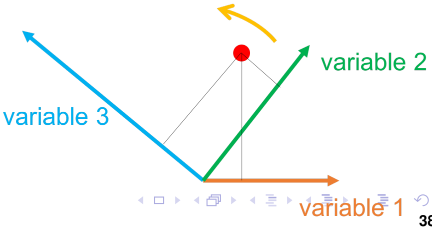
$$\begin{pmatrix} v_{t,1}^{(k)} \\ v_{t,2}^{(k)} \end{pmatrix} \sim \mathrm{N}_2(0, \sigma_k^2 I),$$

Observation model

$$y_{t,j} = \sum_{k=1}^{K} (c_{jk,1} x_{t,1}^{(k)} + c_{jk,2} x_{t,2}^{(k)}) + w_{t,j}$$

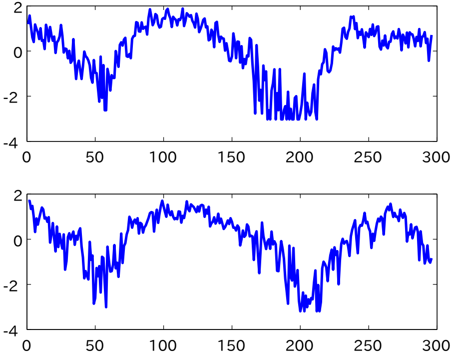
$$(w_{t,1}, \cdots, w_{t,J})^\top \sim \mathrm{N}_J(0, \tau^2 I)$$

- $y_{t,j}$ = sum of inner products of $x_t^{(k)}$ and $c_{jk} = (c_{jk,1}, c_{jk,2})$
  - $c_{1k} = (1, 0)$ for identifiablity
- length and angle of c_{jk} describe amplitude and phase modulation
  - e.g., shorter length for farther sensors



Application to north/south sunspot data

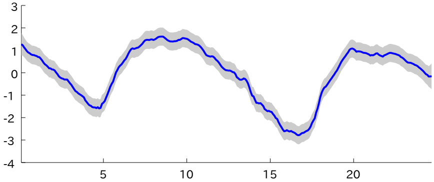
- The number of sunspots on northern/southern hemisphere
  - from ICSU World Data System
  - monthly data from Jan. 1992 to Aug. 2016 (data length: 296)



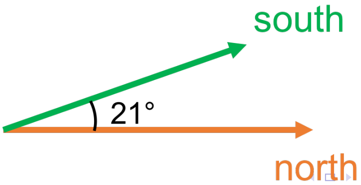
- South sunspot numbers seem to have a slight delay compared to north sunspot numbers.

Phase delay in north/south sunspot data

- Our method decomposes this data into six oscillators.
- The oscillator with 11.76 years period is dominant in power.
  - It corresponds well to the well-known period in sunspot numbers.

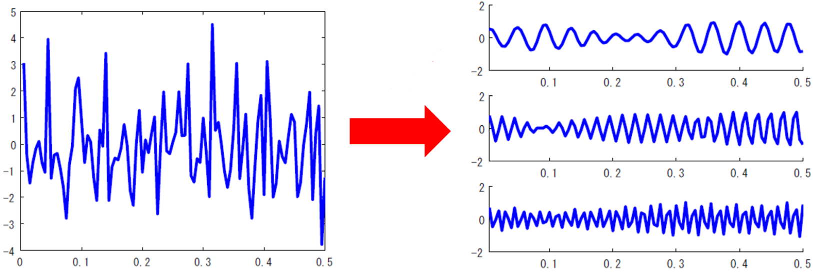


- Projection vector: $(1, 0)$ for north and $(0.82, 0.32)$ for south
  $\rightarrow$ south follows north with a delay of 0.69 years !!



Summary

- We developed a method for decomposing time series data into oscillators (without band-pass filtering).



References

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