Theory of Stochastic Processes
6. Review

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Handouts & Announcements

Handouts

- Slides (this one)

About the midterm exam (reminder + additional note)

- The midterm exam is on May 25 (Thu) in class.
- The exam is open-book and open-note: You can bring any book, note, printed copy and so on. Computers are not allowed.
- It will consist of 4 or 5 questions and will cover material up to May 11.
- The time allowed is 90 minutes.
- Write your answer in English. Grammatical mistakes will not reduce your marks.

Solutions to the remaining exercises

I will use blackboard.

Exercise 1

Show that the symmetric simple random walk is null persistent.

$$\cdots \leftrightarrows \bigcirc \leftrightarrows \bigcirc \leftrightarrows \bigcirc \leftrightarrows \bigcirc \leftrightarrows \cdots$$

$$(p = q = 1/2)$$

Exercise 2

Let $\mathbf{P} = \begin{pmatrix} 0 & 2/3 & 1/3 \\ 1 & 0 & 0 \\ 4/5 & 1/5 & 0 \end{pmatrix}$.

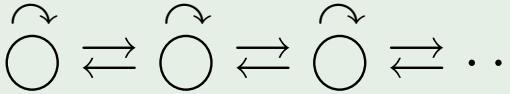
1. Find the stationary distribution $\boldsymbol{\pi}$.
2. Obtain the mean recurrence time $\mu_i = 1/\pi_i$.
3. Calculate μ_i by the definition.

Exercise 3

Let $S = \{1, 2, \cdots\}$ and $\mathbf{P} = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 & \cdots \\ 1/3 & 1/3 & 1/3 & 0 & \cdots \\ 1/4 & 1/4 & 1/4 & 1/4 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$.

Find the stationary distribution $\boldsymbol{\pi}$ if it exists.

Exercise 4 (birth-death process)



$$\mathbf{G} = \begin{pmatrix} -\lambda_0 & \lambda_0 & 0 & 0 & \cdots \\ \mu_1 & -(\lambda_1 + \mu_1) & \lambda_1 & 0 & \cdots \\ 0 & \mu_2 & -(\lambda_2 + \mu_2) & \lambda_2 & \\ \vdots & & \ddots & \ddots & \ddots \end{pmatrix}$$

Is there a stationary distribution?