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THE AXIOMS FOR PLANE GEOMETRY AND PYTHAGORAS' THEOREM

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1. Introduction

In the past several thousand years, many researchers from various countries have reported a lot of properties of plane figures. Many of these are confirmed by calculation of special case or some experiments but few are proved mathematically. In this talk, we propose axioms for plane geometry that are sufficient to extract proofs of theorems which have not been given a proof. And we prove Pythagoras' theorem with mathematical accuracy as an example.

Of course, this is a fictional report as an example.

2. Main Theorem

First of all, we introduce some definitions of conceptions for geometry: such as, *a point is an object without any quantity, a line is a length without width,* etc... see [1] for more details.

In the proof of Pythagoras' theorem we use some lemmas. Lemma 1 is one of them.

Lemma 1. *Let a line p be parallel to a line AB and let points C and D be on the line p. Then dimensions of $\triangle ABC$ and $\triangle ABD$ are equal.*

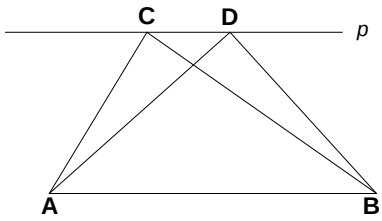
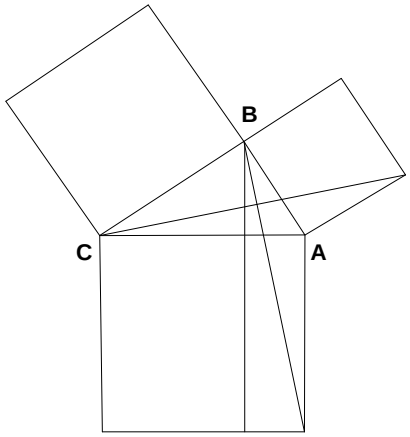


Figure 1. Dimensions of two triangles are equal.

Main matter of this talk is to prove following Pythagoras' theorem using our axioms.

Theorem 1. *Let $\triangle ABC$ be a right triangle with $AB \perp BC$. Then the following equation holds:*

$$AB^2 + BC^2 \quad = \quad AC^2$$



$$\begin{aligned} AB &\perp BC \\ AB^2 + BC^2 &= AC^2 \end{aligned}$$

FIGURE 2. Pythagoras' theorem

3. Conclusion

In this talk, we focus on the proof of Pythagoras' theorem but many other famous properties of figures are also proved with our axioms. Our axioms can be used not only for 2-dimensional figures but also 3-dimensional figures.

We think that our fifth axiom (or parallel postulate) seems rather otiose. We tried to drive the axiom from the others but nothing comes out of our effort. It might sound funny, but there may be the geometry which denies the parallel postulate.

References

[1] T. L. Heath *The Thirteen Books of Euclid's Elements*. University Press, Cambridge, 1926.

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