2nd International Symposium on
Information Geometry and its Applications
December 12-16, 2005, Tokyo

DECISION GEOMETRY

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A decision problem is defined in terms of an outcome space, an action space, and a loss function. Starting from these simple ingredients, we can construct, in a natural way:

(1) A proper scoring rule, for eliciting a probability distribution over the outcome space
(2) An entropy function, measuring the uncertainty in such a distribution
(3) A divergence function, between two distributions
(4) A Riemannian metric, on the space of distributions
(5) A dependence measure, between two random variables
(6) An unbiased estimating equation, for any parametric family of distributions

There is thus a natural geometry associated with any decision problem. Using it, and associated duality properties, we can define such concepts as the "generalised exponential family" associated with a decision problem. There are also connections between the geometry and the statistical game: in particular, between Pythagorean relationships and the existence of a saddle-point.

For the case that the outcome space is itself a Riemannian manifold, can we can define an interesting proper scoring rule, based on the generalised Laplacian, that can be used even when we cannot calculate the normalising constant for a distribution (this is joint work with Steffen Lauritzen).

I am aware that my own grounding in information geometry is very flimsy. My aim in presenting his material is to spark some interest in the problem area, and to learn from the experts how it relates to established theory and how it might be developed.

REFERENCE:

Grunwald, P. D. and Dawid, A. P. (2004). Game theory, maximum entropy, minimum discrepancy, and robust Bayesian decision theory. Ann. Statist. 32, 1367–1433.