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QUANTUM FISHER INFORMATION AND UNCERTAINTY PRINCIPLE

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The noncommutativity in quantum probability has far-reaching consequences. One of the most important is certainly the Heisenberg uncertainty principle

$$\mathrm{Var}_\rho(A)\cdot \mathrm{Var}_\rho(B) \geq \frac{1}{4}|\mathrm{Tr}(\rho[A,B])|^2$$

which is a consequence of SchrNodinger uncertainty principle

$$\mathrm{Var}_\rho(A)\cdot \mathrm{Var}_\rho(B) - |\mathrm{Re}\{\mathrm{Cov}_\rho(A,B)\}|^2 \geq \frac{1}{4}|\mathrm{Tr}(\rho[A,B])|^2.$$

Recently S. Luo and Q. Zhang proved a different kind of uncertainty principle (see [3], Theorem 2), in the SchrNodinger form, where the lower bound appears because the variables A, B are not commuting with the state ρ (in contrast with standard uncertainty principle where the bound depends on the commutator $[A, B]$). This result was conjectured by S. Luo himself and Z. Zhang in a previous paper [4].

The inequality by Luo and Zhang has been recently generalized by Kosaki [2] and Yanagi-Furuichi- Kuriyama [5] and the final result is

$$\mathrm{Var}_\rho(A)\cdot\mathrm{Var}_\rho(B)-|\mathrm{Re}\{\mathrm{Cov}_\rho(A,B)\}|^2 \geq I_{\rho,\alpha}(A)I_{\rho,\alpha}(B)-|\mathrm{Re}\{\mathrm{Corr}_{\rho,\alpha}(A,B)\}|^2 \ \ \forall\alpha\in(0,1)$$

where I and Corr are related to the Wigner-Yanase-Dyson skew information of parameter α.

The purpose of this talk is to put the above inequality in a more geometric form by means of quantum Fisher information (namely the monotone metrics classified by Petz). In this way the lower bound will appear as a simple function of the area spanned by the commutators $i[A,\rho], i[B,\rho]$ in the tangent space to the state ρ, provided the state space is equipped with a suitable monotone metric.

At this point it is natural to ask whether such an inequality holds for every quantum Fisher information. The answer turns out to be negative and a counterexample will be given (see [1]). This is a joint work with T.Isola.

References

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