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WHAT IS THE GEOMETRIC STRUCTURE OF PROBABILITY DENSITY MODELS OF CANONICAL TRAINING SETS FOR CURVED KOHONEN LAYERS?

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The Self-Organizing Map (SOM) is generally known as a method for mapping high dimensional data vectors to a lower dimensional space with topology preservation. SOM approaches have almost exclusively used the Euclidean metric for connecting neighboring cells in the Kohonen layer. Relaxing this constraint is fairly straight forward. It opens up questions about the geometric structure of the probability density models of the training set (TS) that canonically corresponds to the curved Kohonen metric. The idea to consider a curved Kohonen layer was inspired by the prominent cortical fold in the human primary visual cortex (V1), i. e., the calcarine sulcus. Does the shape of V1 influence the cortical magnification factor (CMF) and receptive field size? To answer this questions, we investigated approximations of the retino-cortical map. Approximations can be derived from coefficients of the Jacobian matrix $J = (\frac{\partial x^i}{\partial \phi^j})$. Experiments often characterize only $J_{11} = (\frac{\partial x^1}{\partial \phi^1})$ along a limited number of meridians (ϕ^2), e.g. alongthe vertical and horizontal meridian, called linear CMF. Extra constraints must be applied to determine the map. Usual constrains are isotropy and, alternatively, meridonally symmetry. Given these, a flat approximation of the map can be constructed. The flatness is, however, a further convenient assumption rather than a result. We let V1 assume its typical curved shape and preserve certain metrical measures from the flat approximation of the map, such as surface area or isometry along certain coordinate lines. Depending on the preserved measure, we can build a curved Kohonen layer. Then, in contrast to the standard application of the SOM, we generate a canonical TS (CTS) for this Kohonen layer by choosing a uniform cortical density function on the curved surface and transform it into the retinal domain (ϕ^i). This procedure is consistent with the idea that CMF is proportional to the retinal ganglion cell density. The curved Kohonen layer is trained with its own CTS, and also with a CTS of a flat V1.

As expected, the curved SOM (CSOM) maps its CTS such that in each Voronoi cell of a Kohonen node lies an equal number of data points. For example, if the cortical representation of the horizon is extended larger than other meridian representations due to the process of folding, a central band of smaller receptive fields is observed. This band is a natural consequence of increased retinal CTS density (also known as a visual streak). But even with a meridonally symmetric TS, a common CTS of a flat V1, we find a central band of smaller receptive fields in a curved Kohonen layer. In this case, the CMF is not proportional to TS density throughout the visual field, i. e., the Voronoi cells in the central band consist of less data points.

Our results do not yet contribute to but seek answers from Information Geometry: What is the geometric structure of the CTS? In a simple case, the flat CTS can be chosen meridonally symmetric. Then the joint distribution (ϕ^1, ϕ^2) can be determined from the marginal distributions, in other words the corresponding random variables are independent. On the contrary, a CSOM usually introduces a strong dependency by the visual streak in the CTS. If the geometric structure of the statistical models relates to the geometric structure of the Kohonen layer, a CSOM might dynamically adapt its curvature during the learning phase.