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INFINITE DIMENSIONAL EXPONENTIAL FAMILIES BY REPRODUCING KERNEL HILBERT SPACES

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A new method of constructing infinite dimensional exponential families is proposed. It is shown that, given a positive definite kernel, an exponential family can be defined as a Hilbert manifold which has coordinate neighborhoods given by open sets in the reproducing kernel Hilbert space (RKHS). A useful property of this construction is that the empirical mean parameter for finite samples is well defined as an element of the RKHS owing to the fact that evaluating a function at a point is a continuous functional on the RKHS. It is shown that the empirical mean parameter converges to the population mean parameter in the order $\mathrm{O}_p(n^{1/2})$. Also, a method of pseudo maximum likelihood estimation (pseudo-MLE) with the exponential manifold is introduced as an extension of the ordinary MLE with an exponential family of finite dimension. Using the $n^{1/2}$-consistency of the mean parameter, the consistency of the pseudo-MLE with the exponential manifold is proved.