2nd International Symposium on
Information Geometry and its Applications
December 12-16, 2005, Tokyo

EULER CHARACTERISTIC HEURISTIC FOR APPROXIMATING THE DISTRIBUTION OF THE LARGEST EIGENVALUE OF AN ORTHOGONALLY INVARIANT RANDOM MATRIX

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The Euler characteristic heuristic has been proposed as a method for approximating the upper tail probability of the maximum of a random field with smooth sample path. When the random field is Gaussian, this method is proved to be valid in the sense that the relative approximation error is exponentially smaller. However, very little is known about the validly of the method when the random field is non-Gaussian.

In this talk, as a milestone to developing the general theory about the validity of the Euler characteristic heuristic, we examine the Euler characteristic heuristic for approximating the distribution of the largest eigenvalue of an orthogonally invariant non-Gaussian random matrix. In this particular example, if the probability density function of the random matrix converges to zero sufficiently fast at the the boundary of its support, the approximation error of the Euler characteristic heuristic is proved to be small and the approximation is valid.

Moreover, for several standard orthogonally invariant random matrices, the approximation formula for the distribution of the largest eigenvalue and its asymptotic error are obtained explicitly. Our formulas are practical enough for the purpose of numerical calculations.