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ON A STATISTICALLY EQUIAFFINE MODEL WHICH ISN'T CONJUGATE SYMMETRIC

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The Jeffreys prior, which plays an important role in statistical inference, is generalized to the α-parallel prior by J. Takeuchi and S. Amari[1] from the information geometrical point of view. The α-parallel prior coinsides with the Jeffreys prior if $\alpha = 0$. However the α-parallel prior doesn't always exist for the general value of α ($\alpha \neq 0$). J. Takeuchi and S. Amari give the sufficient condition for the existence of the α-parallel prior such that, if a statistical model is conjugate symmetric, the model is staistically equiaffine. Then they leave the open problem such that whether there exist a statistically equiaffine natural model which is not conjugate symmetric or not. Here we give a solution for their open problem explicitly by considering the q-normal distribution, that belongs to a non-exponential family. The q-normal distribution can smoothly connect the various probability density functions. According as the parameter q changes from $-\infty$ to 3, the q-normal distribution changes from the uniform distribution with a compact support to the uniform distribution with non-compact support, which height is completely 0, via Cauchy distribution, t-distribution and the normal distribution etc. Explict calculations show that the q-normal distribution is not conjugate symmetric, that is to say, α-Riemann tensor isn't antisymmetric for its two of four subscript indices, but the q-normal distribution is statistically equiaffine, that means α-Ricci tensor is symmetric for its two subscript indices. Therefore the answer is that there exists such a model.

References

[1] J. Takeuchi and S. Amari, α-Parallel Prior and Its Properties, *IEEE transactions on Inform. Theory*, Vol. 51, No. 3, pp. 1011-1023, March 2005.