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REVERSE ESTIMATION THEORY, MONOTONE DISTANCES, AND RLD BASED GEOMETRY

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There are two separate approaches in discussing measures which quantifies objects (channels, distance measures, correlations, etc). One is axiomatic approach, the other is operational approach. In recent years, in quantum information theory, an interesting interplay between these two had been discussed: intrude inverse process of some information processing. Then, a 'thermodynamical' argument using a cycle shows that a measure satisfies a sets of axioms takes value between two quantities with key operational meanings.

In this talk, we apply such argument to quantum estimation theory, by introducing "reverse estimation".

The motivation is to find a link between following known observations.

First, a monotone metric takes value between SLD and RLD Fisher metric. This is quite analogous to the situation in other fields of quantum information theory, where a measure with certain properties are sandwiched by two important quantities.

Second, the operational meaning of SLD is the maximal classical Fisher information which can be extracted by measurement from a quantum state family with 1-dim parameter. On the other hand, no such clear interpretation of RLD Fisher metric was known.

Third, SLD add RLD are mutually complement via purification of density matrices, but its operational meaning was not clear.

To find a link between these observations, we define reverse estimation problem, or simulation of quantum state family by probability distribution family. We prove that RLD Fisher metric is a solution to local reverse estimation problem of quantum state family with one-dimensional parameter. Intuitively, this fact is a natural consequence of complementarity with SLD. We can prove that in general, a quantum state family with multi-dimensional parameter is a complement of local estimation, and can be reduced to it.

These results on reverse estimation theory gives new proofs of the follwoing known facts: monotonicity of RLD and SLD Fisher metric, the former is larger than the latter, and a monotone metric is sandwiched by them. Also, we prove a following new fact: a monotone quantum relative entropy is upperbounded by a natural quantum version of relative entropy, which is a path divergence based on RLD metric.

Is that RLD-based relative entropy interpreted as a canonical divergence? To answer the question, we introduce e-connections based on RLD: there can be two natural extension of e-connection, one is based on e-representation of a tangent vector, the other is based on duality with m-connection. In the former, its curvature does not vanish, and the torsion of the latter does not vanish. But, there is a e- and m-autoparallel submanifold of quantum state family such that these two e-connections coincide and its torsion vanishes. Restricted on such submanifold, we can define a cannonical divergence. Notable difference from path-dependent divergence is that our submanifold is autoparallel in both connections.

On such submanifold, the optimal operation for 1-dim local reverse estimation is also optimal operation for global reverse estimation, and if a submanifold satisfies such property, it has the geometrical properties stated before. Also, this kind of submanifold is a complement of Nagaoka's quasi-classical manifold, which is characterized by SLD-based Uhlmann's fiber bundle structure. Naturally, Fujiwara's RLD version of Uhlmann's geometry, being complement to Uhmann's geometry, gives a characterization of such submanifold.

Now, we point out that application of similar approach to classical estimation problem may give a new proof Chenzof's theorem, which states Fisher metric is a unique monotone metric. Different from the existing proof, this new proof, if it goes through, does not explicitely rely on discreteness of support of probability distributions. The key argument is that so-called 'assymptotic sufficiency' and 'local asymptotic normality' gives a way to simulate given statistical model with gaussian distribution family. In this case, two quantities which sandowiches a monotone metrics coinsides, proving uniqueness theorem.

Going back to quantum case, we point out that thermal gaussian state family is a 'standard form', or is asymptotically equivalent to any quantum state family, just as classical gaussian distribution family. Due to this fact, we apply our result to thermal gaussian state family, and proves that P-representation corresponds to the optimal solution to reverse estimation problem. Hence, intuitively, gap between upperbound and lowerbound to monotone metric is essentially due to quantum noise in coherent states.