

REFERENTIAL DUALITY AND REPRESENTATIONAL DUALITY ON STATISTICAL MANIFOLDS

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From the fundamental inequality of a convex function, a family of divergence measures can be constructed, with the convex mixture parameter λ as its index. Such measures, which specialize to α -divergence, Bregman divergence, Jensen difference, etc., can be shown to induce the statistical manifold with a Riemannian metric and a family of torsion-free connections indexed by λ . We use this construction to derive generalized expressions for the Fisher metric and α -connections, both in the finite-dimensional and infinite-dimensional settings, with or without positivity and normalization restrictions associated with probability manifolds. A clear distinction is drawn between two senses of duality on a statistical manifold, namely, "referential duality" regarding the effect of assignment, in the divergence measure, reference/comparison status of the two (probability density) functions, and "representational duality" regarding the effect of a monotonic transformation of these functions on the induced Riemannian structure (metric and conjugate connections).