2nd International Symposium on
Information Geometry and its Applications
December 12-16, 2005, Tokyo

AN APPLICATION OF THE DIFFERENTIAL GEOMETRICAL MANIFOLD STRUCTURE MODELED ON THE TOPOLOGY OF ORLICZ SPACES FOR THE STUDY OF LOCAL MARTINGALES

DANIELE IMPARATO

Department of Mathematics, Politecnico of Turin, Italy
E-mail address: daniele.imparato@polito.it

Let $(\Omega, \mathcal{F}, \mu, (\mathcal{F}_t)_{t\geq 0})$ be a filtered probability space satisfying the usual conditions; let us denote by $L^{\Phi_1}(\mu)$ the Orlicz space associated with the Young function $\Phi_1(x) := \cosh(x) - 1$ and by $L^{2,\Phi_1}(\mu)$ the space of r.v. whose square belongs to $L^{\Phi_1}(\mu)$.

Pistone and Sempi (1995) and Pistone and Rogantin (1999) constructed a differential geometrical manifold structure locally modeled on the topology of $L_0^{\Phi_1}(\mu)$ - sub-0 denotes centered r.v.- for the set of μ-almost surely strictly positive densities; this work is connected to the theory of information geometry - see Amari and Nagaoka (2000) for details.

Our work deals with the application of such a mathematical framework to martingales theory and it is part of the research for the authorfs Phd thesis in Mathematics, supervisor G. Pistone (Politecnico of Turin). As an example, the results can be developed for setting up financial problems within incomplete markets, aimed at choosing the optimal equivalent martingale measure. For general references, see Frittelli (2000), Gao (2002-2004), and Bellini and Frittelli (2002).

We show the continuity of the product

$$\begin{array}{rcl} L^{2,\Phi_1}(\mu) \times L^{2,\Phi_1}(\mu) & \rightarrow & L^{\Phi_1}(\mu) \\ (u,v) & \mapsto & uv \end{array}$$

as a bilinear form.

We endow the set of equivalent martingale measures with the previous geometrical structure. The main result is the proof of the following statement.

Theorem . *Let* $(M_t),\ (N_t) \in L^{\Phi_1}(\mu),\ t \in [0,T]$, *be two local martingales with continuous trajectories and* τ *be a bounded stopping time. Then there exists a constant* $k > 0$ *such that*

$$\|\langle M, N\rangle_\tau\|_{(\Phi_1,\mu)} \leq k \|M_\tau\|_{(\Phi_1,\mu)} \|N_\tau\|_{(\Phi_1,\mu)},$$

i.e. the crochet $\langle \cdot, \cdot \rangle$ *is a continuous bilinear form in* $L^{\Phi_1}(\mu)$.