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INFORMATION GEOMETRY OF QUANTUM CHANNEL ESTIMATION

AKIO FUJIWARA

Department of Mathematics, Osaka University

I review some recent developments in estimation theory for a smooth parametric family $\{\Gamma_\theta\,;\,\theta = (\theta^1, ..., \theta^d) \in \Theta\}$ of quantum channels (= trace preserving completely positive maps) acting on the set $\mathcal{S}(\mathcal{H})$ of density operators on a Hilbert space $\mathcal{H}$ [1]. In particular, I will focus on the extension $(\mathrm{id} \otimes \Gamma_\theta)^{\otimes n} : \mathcal{S}((\mathcal{H} \otimes \mathcal{H})^{\otimes n}) \to \mathcal{S}((\mathcal{H} \otimes \mathcal{H})^{\otimes n})$, where n is an arbitrary positive integer, and will clarify the role of quantum entanglement and the degree n of extension, putting emphasis on an active interplay between noncommutative statistics [2] and information geometry [3].

In view of the quantum Cramér-Rao type estimation theory, there are at least two classes of quantum channels that exhibit essentially different asymptotic behaviors: the minimum estimation error is of $O\left(1/n\right)$ for generalized Pauli channels [4] as is usually the case in classical statistics, whereas it is of $O\left(1/n^2\right)$ for $SU(d)$ channels [5] [6]. The underlying geometrical mechanism behind these behaviors is that the degree α of entanglement controls the "shape" of the manifold $\{\Gamma_\theta\,;\,\theta \in \Theta\}$ embedded in the state space $\mathcal{S}((\mathcal{H} \otimes \mathcal{H})^{\otimes n})$, while the degree n of extension controls its "radius." It is an open question whether there are classes of quantum channels that exhibit different asymptotic rates $O\left(1/n^s\right)$ with $s \neq 1, 2$. Nevertheless, the present study demonstrates the usefulness of differential geometrical methods in quantum channel estimation theory.

References

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