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KULLBACK DIVERGENCE IN GAME-THEORETIC PROBABILITY PROTOCOLS

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In the framework of the game-theoretic probability of Shafer and Vovk (2001), we study capital process behavior in general linear protocols. We show that if Skeptic uses a Bayesian strategy with appropriate prior, the capital process is explicitly described in terms of the past average of Reality's moves. From this it is proved that the Skeptic's Bayesian strategy weakly forces strong law of large numbers (SLLN) with the convergence rate of $O(\sqrt{\log n/n})$. Furthermore if Reality violates SLLN then the exponential growth rate of the capital process is very accurately described in terms of the Kullback divergence between the average of Reality's moves when she violates SLLN and the average when she observes SLLN. This approach provides a justification of Kullback divergence without any probabilistic assumptions.